late

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Problem 4

Show $[S^{\mu\nu}, S^{\rho\sigma}] = i[g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}]$

$$[S^{\mu\nu}, S^{\rho\sigma}] = \left(\tfrac{i}{4}\right)^2 [[\mu,\nu],[\rho,\sigma]] = \left(\tfrac{i}{4}\right)^2 [\mu\nu - \nu\mu, \rho\sigma - \sigma\rho]$$

$$= \left(\tfrac{i}{4}\right)^2 \{ [\mu\nu, \rho\sigma] - [\nu\mu, \rho\sigma] - [\mu\nu, \sigma\rho] + [\nu\mu, \sigma\rho] \}$$

$$= \left(\tfrac{i}{4}\right)^2 \{ \mu\nu\rho\sigma - \rho\sigma\mu\nu - \nu\mu\rho\sigma + \rho\sigma\nu\mu - \mu\nu\sigma\rho + \sigma\rho\mu\nu + \nu\mu\sigma\rho - \sigma\rho\nu\mu \}$$

$$= \left(\tfrac{i}{4}\right)^2 \left\{ \begin{array}{llll} A & B & C & \\ \cancel{\mu\nu\rho\sigma} & -\cancel{\mu\nu\rho\sigma} & +\cancel{\mu\nu\rho\sigma} & 1 \\ \cancel{\rho\sigma\mu\nu} & -\cancel{\rho\sigma\mu\nu} & -\cancel{\rho\sigma\mu\nu} & 2 \\ \cancel{\nu\mu\rho\sigma} & -\cancel{\nu\mu\rho\sigma} & -\cancel{\nu\mu\rho\sigma} & 3 \\ \cancel{\rho\sigma\nu\mu} & -\cancel{\rho\sigma\nu\mu} & +\cancel{\rho\sigma\nu\mu} & 4 \\ \mu\nu\sigma\rho & -\cancel{\mu\nu\sigma\rho} & -\cancel{\mu\nu\sigma\rho} & 5 \\ \cancel{\sigma\rho\mu\nu} & -\cancel{\sigma\rho\mu\nu} & +\cancel{\sigma\rho\mu\nu} & 6 \\ \cancel{\nu\mu\sigma\rho} & -\nu\mu\sigma\rho & +\cancel{\nu\mu\sigma\rho} & 7 \\ \cancel{\sigma\rho\nu\mu} & -\cancel{\sigma\rho\nu\mu} & -\cancel{\sigma\rho\nu\mu} & 8 \end{array} \right\}$$

$$i[g^{\nu\rho} S^{\mu\sigma} - g^{\mu\rho} S^{\nu\sigma} - g^{\nu\sigma} S^{\mu\rho} + g^{\mu\sigma} S^{\nu\rho}]$$

$$= \frac{(i)^2}{8} \left(\{\nu,\rho\}[\mu,\sigma] - \{\mu,\rho\}[\nu,\sigma] - \{\nu,\sigma\}[\mu,\rho] + \{\mu,\sigma\}[\nu,\rho] \right)$$

$$= \frac{i^2}{8} \left((\nu\rho + \rho\nu)(\mu\sigma - \sigma\mu) - (\mu\rho + \rho\mu)(\nu\sigma - \sigma\nu) - (\nu\sigma + \sigma\nu)(\mu\rho - \rho\mu) + (\mu\sigma + \sigma\mu)(\nu\rho - \rho\nu) \right)$$

$$= \frac{i^2}{8} [\nu\rho\mu\sigma - \nu\rho\sigma\mu + \rho\nu\mu\sigma - \rho\nu\sigma\mu - \mu\rho\nu\sigma + \mu\rho\sigma\nu - \rho\mu\nu\sigma + \rho\mu\sigma\nu$$
$$- \nu\sigma\mu\rho + \nu\sigma\rho\mu - \sigma\nu\mu\rho + \sigma\nu\rho\mu + \mu\sigma\nu\rho - \mu\sigma\rho\nu + \sigma\mu\nu\rho - \sigma\mu\rho\nu]$$

$\frac{i^2}{8}\nu\rho\mu\sigma = B3 + C3$

$-\frac{i^2}{8}\nu\rho\sigma\mu = A8 + B4$

$\frac{i^2}{8}\rho\nu\mu\sigma = 1$

$-\frac{i^2}{8}\rho\nu\sigma\mu = A4 + C4$

$-\frac{i^2}{8}\mu\rho\nu\sigma = A1 + C1$

$+\frac{i^2}{8}\mu\rho\sigma\nu = A5 + B7$

$\frac{i^2}{8}\rho\mu\nu\sigma = B6 + A2$

$+\frac{i^2}{8}\rho\mu\sigma\nu = B2 + C2$

$-\frac{i^2}{8}\nu\sigma\mu\rho = A7 + C7$

$+\frac{i^2}{8}\nu\sigma\rho\mu = 1$

$-\frac{i^2}{8}\sigma\nu\mu\rho = B8 + C8$

$+\frac{i^2}{8}\sigma\nu\rho\mu = \quad + \quad 2$

$+\frac{i^2}{8}\mu\sigma\nu\rho = B5 + C5$

$-\frac{i^2}{8}\mu\sigma\rho\nu = 2$

$+\frac{i^2}{8}\sigma\mu\nu\rho = B1 + A3$

$-\frac{i^2}{8}\sigma\mu\rho\nu = A6 + C6$ term you want.

where?

I guess I don't understand: from top $B3 + C3 = -2\nu\mu\rho\sigma$

$= 2\nu^{\rho}\mu\sigma - 4\nu\sigma g^{\mu\rho}$

Lastly, the two terms labelled 1 and 2 sum to zero: ?

$+2g^{\rho\nu}{}_{\mu\sigma}$ $\quad$ $+2g^{\rho\nu}{}_{\mu\sigma} - 2g^{\rho\mu}{}_{\nu\sigma}$ etc.

$$\rho\nu\mu\sigma = -\nu\rho\mu\sigma = \nu\mu\rho\sigma = -\nu\mu\sigma\rho = \nu\sigma\mu\rho$$

$$= -\nu\sigma\rho\mu \qquad (\text{so } \rho\nu\mu\sigma + \nu\sigma\rho\mu = 0)$$

and

$$\sigma\nu\rho\mu = -\sigma\nu\mu\rho = \sigma\mu\nu\rho = -\mu\sigma\nu\rho = \mu\sigma\rho\nu$$

$$(\text{so } \sigma\nu\rho\mu - \mu\sigma\rho\nu = 0)$$

Since you can always add zero to something without changing it, I've accounted for all the terms in $i[g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}]$ and showed they can be combined to form $[S^{\mu\nu}, S^{\rho\sigma}]$.