Divide space up into pos + neg freq components $\phi = \phi^+(x) + \phi^-(x)$

$$\phi^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_p e^{-ip\cdot x} + \cdots a_p^\dagger e^{+ip\cdot x}$$

$$T[\phi(x)\phi(y)] \overset{x^0>y^0}{=} \phi^+(x)\phi^+(y) + \phi^+(x)\phi^-(x) + \underline{\phi^-(x)\phi^+(y)} + \phi^-(x)\phi^-(y)$$ time ordering

But we'd like to have all the plus guys (guys who annhilate vacuum) to the right. There is one guy to worry about. We can rewrite that term as:

$$\phi^-(x)\phi^+(y) = \phi^-(y)\phi^+(x) + [\phi^+(x), \phi^-(y)]$$
$$= \phi^-(y)\phi^+(x) + \overbrace{\phi(x)\phi(y)}$$ "contraction"

So we can rewrite $T[\phi(x)\phi(y)] = N[\phi(x)\phi(y)] + \overbrace{\phi(x)\phi(y)}$.

$$\boxed{\overbrace{\phi(x)\phi(y)} = \begin{cases} [\phi^+(x), \phi^-(y)] & x^0 > y^0 \\ [\phi^+(y), \phi^-(x)] & y^0 > x^0 \end{cases}}$$

Since $[\phi(x), \phi(y)]$ involves commutators of a_p and $a_p^\dagger$, these contractions are c numbers and therefore we can write:

$$\overbrace{\phi(x)\phi(y)} = \langle 0|\overbrace{\phi(x)\phi(y)}|0\rangle \overset{\text{to prove}}{=} \langle 0|T\phi(x)\phi(y)|0\rangle = D_F(x-y)$$

We'll prove this last statement for $x^0 > y^0$. But ϕ^+ has a_p and ϕ^- has $a_p^\dagger$. Thus $\phi^+|0\rangle = 0$.

$$\langle 0|T\phi(x)\phi(y)|0\rangle = \langle 0|(\phi^+(x) + \phi^-(x))(\phi^+(y) + \phi^-(y))|0\rangle$$
$$= \langle 0|\phi^+(x)\phi^-(y)|0\rangle$$
$$= \langle 0|\phi^+(x)\phi^-(y) - \phi^-(y)\phi^+(x)|0\rangle$$ since $\phi^-(y)\phi^+(x)|0\rangle = 0$ (adding zero)
$$= [\phi^+(x), \phi^-(y)]$$

since these are c-numbers. So we just proved half of:

$$\hat{T}[\phi(x)\phi(y)] = \hat{N}[\phi(x)\phi(y)] + \langle 0|T\phi(x)\phi(y)|0\rangle$$

eg: $$\hat{T}\{\phi_1\phi_2\phi_3\phi_4\} = \hat{N}\{\phi_1\phi_2\phi_3\phi_4 + \cdots + \phi_1\overbrace{\phi_2\phi_3}\phi_4 + \phi_1\overbrace{\phi_2\phi_3\phi_4} + \cdots\}$$