Time Reversal

TR changes the time coordinate of a wf: $T\psi(t,\bar{x}) = \psi(-t,\bar{x})$. What operator does this? $a^{\dagger}_{\bar{p}} = a^{\dagger}_{-\bar{p}}$ is good but not enough. It should also be the case that $[T,H]=0$. If you accept this, you can derive:

$$\psi(t,\bar{x}) = e^{iHt}\psi(\bar{x})e^{-iHt}$$

Apply T to both sides. Since T commutes with the hamiltonian,

$$T\psi(t,\bar{x})T = e^{iHt}T\psi(x)e^{-iHt}T$$

$$\underbrace{T\psi(t,\bar{x})T|0\rangle}_{\psi(-t,\bar{x})\text{ contains } b^{\dagger}e^{-iE_p t}} = e^{iHt}\underbrace{T\psi(x)T}_{\text{contains } b^{\dagger}e^{iE_p t}}e^{-iHt}|0\rangle$$

which is in conflict. Therefor T must be anti unitary since it must take $\psi \to \psi^*$. Note that anti-unitary here doesn't mean $\psi^{\dagger} = \psi^{-1}$ it means there's an extra added instruction. T^{op} must include a matrix T <u>and</u> complex conjugation. Note that since a, b have spin and spin $= \bar{r}\otimes\bar{p}$ then $a^{st}_{\bar{p}} \to a^{-st}_{-p}$. After <u>alot</u> of algebra we find

$$T\psi(\bar{x},t) = \underbrace{\gamma^1\gamma^3}_{T}\psi(-t,x)$$

$$T\gamma^{\mu}T = (\Lambda_T^{-1})^{\mu}{}_{\nu}\gamma^{\nu}$$

$$T\gamma^{\mu *}T = \begin{cases} -\gamma^0 \\ +\bar{\gamma} \end{cases}$$

$T = \gamma^1\gamma^3$ satisfies this (excersize to the reader)

Now we can do what we did for the parity case. Look at the bilinears.

① $T^{op}\bar{\psi}(\bar{x},t)T^{op} = (T^{op}\psi T^{op})^{\dagger}\gamma^{0*}$ ← must look at this too)

$\quad = \bar{\psi}(\bar{x},-t)[-\gamma\gamma^3]$

② $T^{op}\bar{\psi}\psi T^{op} = \bar{\psi}(\bar{x},-t)(-\gamma^1\gamma^3)(\gamma^1\gamma^3)\psi(\bar{x},-t) = \bar{\psi}(\bar{x},-t)\psi(\bar{x},-t)$ Behaves like a scalar!!

③ $T^{op}(\bar{\psi}i\gamma^5\psi)T^{op} = -i\bar{\psi}(\bar{x},-t)(-\gamma^1\gamma^3)\gamma^5(\gamma^1\gamma^3)\psi(\bar{x},-t) = -i(\bar{\psi}\gamma^5\psi)$ at $\bar{x},-t$.

④ $T^{op}\bar{\psi}\gamma^{\mu}\psi T^{op}$

$$= \begin{cases} \bar{\psi}\gamma^0\psi \\ -\bar{\psi}\bar{\gamma}\psi \end{cases}$$

← why??? zeroth component is charge and 1,2,3 is current. Physically this is <u>exactly</u> what we expect

recall $\gamma^1\gamma^3\gamma^{\mu *} = \begin{cases} -\gamma^0 \\ \bar{\gamma} \end{cases}$

Charge Conjugation

CC is not part of Lorentz transformations, so it's not necessary that ψ be a representation of C. Let's see if that happens.

$$\left.\begin{aligned} C a_p^s C &= b_p^s \\ C b_p^s C &= a_p^s \end{aligned}\right\} \text{what we expect}$$

What does C do to ψ? It involves alot of algebra which we won't do here.

$V^s(p) = i\gamma^2 U^s(p)^*$ $\quad$ $U^s(p) = i\gamma^2 V^s(p)^*$ $\quad$ then:

$$C\psi(x)C = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s b_p^s e^{-ip\cdot x}\underbrace{i\gamma^2 V^s(p)^*}_{U^s(p)} + a_p^{s\dagger} e^{ip\cdot x}\underbrace{i\gamma^2 U^s(p)^*}_{V^s(p)}$$

what is this??

$$= i\gamma^2\psi^*(x) = i\gamma^2(\psi^\dagger)^T = i\gamma^2(\bar\psi\gamma^0)^T = i(\bar\psi\gamma^0\gamma^2)^T$$

use $\gamma^{2T} = \gamma^2$

$$C\bar\psi C = (i\gamma^0\gamma^2\psi)^T$$

$\therefore$ C turns ψ into $\bar\psi^T$ and $\bar\psi$ into ψ^T with stuff in front so $C_{matrix} = i\gamma^0\gamma^2$

$$C^{op}\bar\psi C^{op} = (i\gamma^0\gamma^2\psi)^T = (C\psi)^T$$
$$C^{op}\psi C^{op} = (i\bar\psi\gamma^0\gamma^2)^T = (\bar\psi C)^T$$

Bilinear Constant: $C^{op}\bar\psi\psi C^{op} = (i\gamma^0\gamma^2\psi)^T(i\bar\psi\gamma^0\gamma^2)^T$

$$= -\psi^T\underbrace{(\gamma^2)^T(\gamma^0)^T(\gamma^2)^T(\gamma^0)^T}_{=+1}\bar\psi^T$$

with $(\gamma^2)^T = \gamma^2$, $(\gamma^0)^T = \gamma^0$, $(\gamma^2)^T = \gamma^2$, $(\gamma^0)^T = \gamma^0$

$$= -\psi^T\bar\psi^T$$
$$= \bar\psi\psi + \text{infinite constant}$$

infinite constant $\to 0$

$:aa^\dagger: \equiv -a^\dagger a$ for fermions

$: \; :$ means all a's and b's are right of $a^\dagger$ and $b^\dagger$

$$C^{op}\, i\bar{\psi}\gamma^5\psi\, C^{op} = i\bar{\psi}\gamma^5\psi$$

$$\begin{aligned}
C^{op}\underbrace{\bar{\psi}\gamma^\mu\psi}_{\substack{\text{this is a current so}\\ \text{we expect a sign change}}}C^{op} &= (i\gamma^0\gamma^2\psi)^T\gamma^\mu(i\bar{\psi}\gamma^0\gamma^2)^T \\
&= -\psi^T\gamma^{2T}\gamma^{0T}\gamma^\mu\gamma^{2T}\gamma^{0T}\bar{\psi}^T \\
&= -\psi^T\underbrace{\gamma^2\gamma^0\gamma^\mu\gamma^2\gamma^0}_{-\gamma^{\mu T}}\bar{\psi}^T \\
&= \psi^T\gamma^{\mu T}\bar{\psi}^T \\
&= -\bar{\psi}\gamma^\mu\psi + \underset{\to 0}{\cancel{\infty}}
\end{aligned}$$

So now we know what CPT does. Very important to particle physics. It was thought that C, P, T ~~hold~~ separately were symmetries of all Lagrangians. However, the weak interaction violates P, T weakly. What happens to all 3 together? CPT together is <u>always</u> a symmetry of all local field theories like QFT. We can write down all possible bilinear forms.

	∂_μ	$\bar{\psi}\psi$	$i\bar{\psi}\gamma^5\psi$	current $\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$
P	$(-1)^\mu$	1	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$
T	$-(-1)^\mu$	1	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$
C	1	1	1	-1	1	-1
	-1	1	1	-1	-1	1

$-(-1)^\mu$: behaves as a pseudo vector

$(-1)^\mu$: " " " vector

note: $\bar{\psi}\gamma^\mu\gamma^5\psi$: generator of chiral transformations which leaves Lagrangian invariant if mass equals zero.

$\bar{\psi}\sigma^{\mu\nu}\psi$: $\sigma^{\mu\nu}$ in low energy limit gives anomalous magnetic moment.

It looks like stuff isn't invariant under CPT! Look at the -1. However, look at interactions. We need to show that $CPT = +1$ $\forall$ lagrangians. $\mathcal{L}$ is a Lorentz scalar. All our scalars above are CPT $+1$. All our vectors are -1 but vector $\cdot$ vector has CPT $+1$. A tricky case is the Dirac eqn itself.

$$\mathcal{L}_0 = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$$

$P^{\varphi} \mathcal{L}_0(t,\bar{x}) P^{\varphi} = \mathcal{L}_0(t,-\bar{x})$ is OK

$T^{\varphi} \mathcal{L}_0(t,\bar{x}) T^{\varphi} =$ tricky but recall $i \to -i$ since T is anti-unitary but we do get $\mathcal{L}_0(-t,\bar{x})$.

$C^{\varphi} \mathcal{L}_0(t,\bar{x}) C^{\varphi} = -(\partial_\mu \bar{\Psi}) i \gamma^\mu \Psi$