$\hbar = c = 1$ $\qquad$ length = time = $\frac{1}{\text{energy}}$ = $\frac{1}{\text{mass}}$

$$(1\,\text{GeV})^{-1} = (10^9\,\text{eV})^{-1} = .197 \times 10^{-13}\,\text{cm} = .197\ \text{fm}$$
$$(1\,\text{GeV})^{-2} = (10^9\,\text{eV})^{-2} = .389 \times 10^{-27}\,\text{cm}^2 = .389\ \text{barn}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$x^\mu = (x_0, \vec{x})$$
$$x_\mu = g_{\mu\nu} x^\nu = (x_0, -\vec{x})$$

$$\vec{p} \cdot \vec{x} = p^\mu x_\mu = p^0 x_0 - \vec{p} \cdot \vec{x} \qquad \text{4 vector contraction}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \vec{\nabla}\right)$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0}, -\vec{\nabla}\right)$$

$$\left\{\begin{array}{l} \varepsilon^{0123} = +1 \\ \varepsilon_{0123} = -1 \end{array}\right.$$

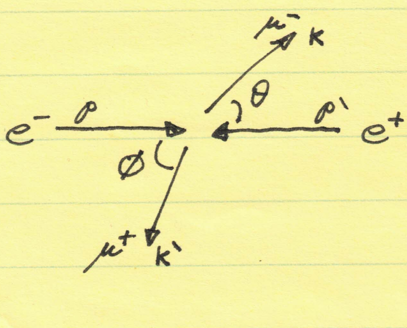
In this convention, $E = i\frac{\partial}{\partial x^0}$, $p = -i\vec{\nabla} \Rightarrow p^\mu = i\partial^\mu$

Heavyside Lorentz convention of E+M : $\Phi = \frac{Q}{4\pi r}$

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{e^2}{4\pi} = \frac{1}{137}$$

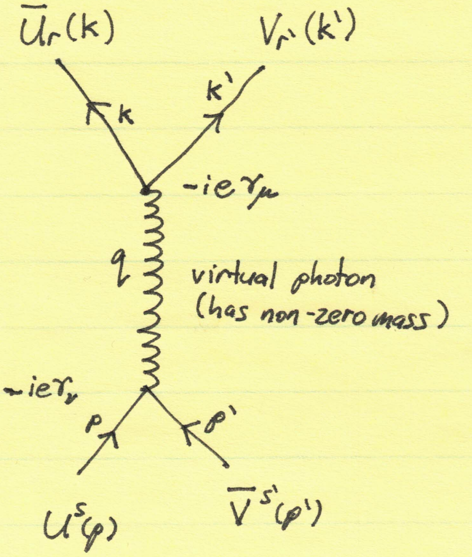
## QED

$e^+e^- \rightarrow \mu^+\mu^-$ $\qquad$ Electric force has to be involved.




$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{64\pi^2 E_{CM}^2} |M|^2$$

$M$ matrix is the whole point – it's what we need to calculate. It must be obtained via perturbation theory.




<u>Momentum conservation</u>

$$k + k' = p + p' = \underset{\text{virtual photon}}{q}$$

Generalized definition of a photon: mediator of the EM force, not just what you see from a lightbulb.

$$\left( \quad \bar{u} \quad \right) \left( \quad \gamma \quad \right) \left( \bar{v} \right)$$

A vertex comes with an algebraic structure.

## <u>Dirac / KG</u>

$E = mc^2$

$\Delta E \, \Delta t \sim \hbar$

Charge is no longer a constant in QFT!

$$u(t) = \langle \vec{x} | e^{-iHt} | \vec{x}_0 \rangle$$

$$= \langle \vec{x} | e^{-i\hat{p}^2/2m \, t} | \vec{x}_0 \rangle$$

Hamiltonian for non-rel free particle

$$= \int d^3p \, \langle \vec{x} | e^{-i\hat{p}^2 t/2m} | \vec{p} \rangle \langle \vec{p} | \vec{x}_0 \rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \, e^{-i\hat{p}^2 t/2m} \, e^{i\vec{p}\cdot(\vec{x}-\vec{x}_0)}$$

$$= \left( \frac{m}{2\pi i t} \right)^{3/2} e^{im(\vec{x}-\vec{x}_0)^2/2t}$$

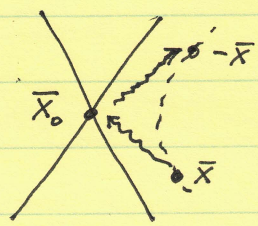
$$\neq 0 \quad \text{for all } t$$

So even if $t=0$ we have influence at $\bar{x}$ from $\bar{x}_0$. This violates causality!

If we use relativistic hamiltonian $H = \sqrt{p^2+m^2}$ then we find

$$U(t) = e^{-m\sqrt{(x-x_0)^2 - t^2}}$$

exponential dampening, but still not zero! Still violates causality.




Particles always have anti-particles. That's how QFT fixes causality.

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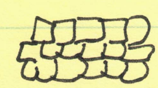
Lagrangian for continuous material: Goldstein, Kaku. The starting point:

$$S = \int L\,dt = \int \mathcal{L}[\underbrace{\phi(x)}_{\text{density of jelly}}, \underbrace{\partial_\mu \phi(x)}_{\text{interaction + KE of jelly}}]\, d^4x$$



picture a glob of jelly

Then by Hamilton's principle,

$$\delta S = \int \left\{ \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta(\partial_\mu \phi) \right\} d^4x$$

$$= \int \left\{ \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right)\delta\phi + \underbrace{\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta\phi\right)}_{\text{surface term}} \right\} d^4x = 0$$

$$\Longrightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

Just like for a particle.

$$H = p\dot{q} - L \qquad p = \frac{\partial L}{\partial \dot{q}}$$

$$p(x) = \frac{\partial L}{\partial \dot{\phi}(x)} = \frac{\partial}{\partial \dot{\phi}(x)} \int \mathcal{L}[\phi(x), \partial_\mu \phi(x)]\, d^3x = \underbrace{\Pi(\bar{x})}_{\frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}} d^3x$$

Generalize the Hamiltonian:

$$H = \int d^3x \left[ \Pi(\bar{x}) \dot{\phi}(x) - \mathcal{L} \right]$$

Lagrangian for the free KG

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\bar{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2$$
$$= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

What is the equation of motion? Apply Lagrange's equation:

$$-m^2\phi - \partial^\mu(\partial_\mu \phi) = 0$$
$$(\partial^\mu \partial_\mu + m^2)\phi = 0$$
$$(\partial_0^2 - \bar{\nabla}^2 + m^2)\phi = 0$$
$$(-E^2 + p^2 + m^2)\phi = 0$$
$$E^2\phi = (p^2 + m^2)\phi$$

$$\Pi(\bar{x}) = \dot{\phi}(\bar{x})$$

canonical conjugate to $\phi(x)$ – momentum density

$$\mathcal{H} = \frac{1}{2}\Pi^2(x) + \frac{1}{2}(\bar{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2$$

## Noether's Theorem

$\mathcal{L}(\bar{x}) \to \mathcal{L}(\bar{x}) + \alpha \partial_\mu J^\mu(\bar{x})$ leaves eom unchanged

$\phi(\bar{x}) \to \phi(x) + \alpha \Delta\phi(x)$ shift coordinate by amount proportional to $\alpha$.

This all happens at a single location.

$$\alpha \Delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi}(\alpha \Delta \phi) + \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right)\partial_\mu(\alpha \Delta \phi)$$

$$= \alpha \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi\right) + \alpha \underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right]}_{=0 \text{ by Lagrange}} \Delta \phi$$

$\uparrow$

so if $\mathcal{L}$ is unchanged, this must be a conserved quantity. (we call it a current for some reason)

$$\partial_\mu J^\mu = \partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi\right]$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu \qquad \boxed{\partial_\mu j^\mu = 0}$$