

Problem 3.62

If a r.v. X is such that $\langle (X-1)^2 \rangle = 10$ and $\langle (X-2)^2 \rangle = 6$, find $\langle X \rangle$, $\text{Var}(X)$, σ_x .

From the given information:

$$\begin{aligned}\langle (X-1)^2 \rangle &= \langle X^2 - 2X + 1 \rangle = \langle X^2 \rangle - 2\langle X \rangle + 1 = 10 \\ \Rightarrow \langle X^2 \rangle - 2\langle X \rangle &= 9 \quad (i)\end{aligned}$$

$$\begin{aligned}\langle (X-2)^2 \rangle &= \langle X^2 - 4X + 4 \rangle = 6 \\ \Rightarrow \langle X^2 \rangle - 4\langle X \rangle &= 2 \quad (ii)\end{aligned}$$

Computing $2 \times (i) - (ii)$:

$$\begin{array}{rcl} 2\langle X^2 \rangle - 4\langle X \rangle & = & 18 \\ - \quad \langle X^2 \rangle - 4\langle X \rangle & = & 2 \\ \hline \end{array} \Rightarrow \underline{\underline{\langle X^2 \rangle = 16}}$$

And plugging this back into eqn (i),

$$16 - 2\langle X \rangle = 9 \Rightarrow \underline{\underline{\langle X \rangle = \frac{7}{2}}}$$

So:

$$\text{Var}(X) = 16 - \left(\frac{7}{2}\right)^2 = \frac{64 - 49}{4} = \frac{15}{4}$$

$$\sigma_x = \frac{\sqrt{15}}{2}$$