Problem 3.59

Let X be a random variable having density function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find Var(X) and σ_X.

As usual, we will compute $\sigma_X^2 = \langle x^2 \rangle - \langle x \rangle^2$.

$$\langle x^2 \rangle = \int_0^\infty x^2 e^{-x}\, dx \qquad \begin{matrix} u = x^2 & du = dx \\ dV = e^{-x}dx & V = -e^{-x} \end{matrix}$$

$$= 2\int_0^\infty x e^{-x}\, dx \qquad \begin{matrix} u = x & du = dx \\ dV = e^{-x}dx & V = -e^{-x} \end{matrix}$$

$$= 2\int_0^\infty e^{-x}\, dx$$

$$= 2e^{-x}\Big|_\infty^0$$

$$= 2$$

$$\langle x \rangle = \int_0^\infty x e^{-x}\, dx \qquad \begin{matrix} u = x & du = dx \\ dV = e^{-x}dx & V = -e^{-x} \end{matrix}$$

$$= \cancel{-xe^{-x}\Big|_0^\infty}^{\,0} + \int_0^\infty e^{-x}\, dx$$

$$= e^{-x}\Big|_\infty^0$$

$$= \underline{1}$$

So:

$$\sigma_X^2 = \langle x^2 \rangle - \langle x \rangle^2 = 2 - 1 = \underline{1}$$

$$\sigma_X = \sqrt{\sigma_X^2} = \underline{1}$$