

Problem 3.58

Let X be a random variable having density function

$$f(x) = \begin{cases} \frac{1}{4} & -2 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $\text{Var}(X)$ and σ_x .

As usual we will compute $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ and $\sigma_x = \sqrt{\sigma_x^2}$.

$$\langle x^2 \rangle = \int_{-2}^2 \frac{1}{4} x^2 dx = \left. \frac{1}{12} x^3 \right|_{-2}^2 = \left. \frac{1}{6} x^3 \right|_0^2 = \frac{4}{3}$$

We know that $\langle x \rangle^2 = \langle x \rangle = 0$ because the integrand, $\frac{1}{4}x$, is an odd function centred on $x=0$. Therefore,

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{4}{3} - 0 = \frac{4}{3}$$

$$\sigma_x = \sqrt{\sigma_x^2} = \frac{2}{\sqrt{3}}$$