Problem 3.44

Let X be a r.v. defined by the density function

$$f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $\langle x \rangle$, $\langle 3x-2 \rangle$, $\langle x^2 \rangle$.

$$\langle x \rangle = \int_{-\infty}^{\infty} x f(x)\,dx = 3\int_0^1 x^3\,dx = \frac{3}{4}x^4\Big|_0^1 = \frac{3}{4}$$

$$\langle 3x-2 \rangle = 3\langle x \rangle - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^1 3x^4\,dx = \frac{3}{5}x^5\Big|_0^1 = \frac{3}{5}$$