

Problem 3.8

Find the variance and standard deviation of the sum obtained from tossing a pair of dice.

Let X and Y be the random variables associated with the event of rolling a 6-sided die. So we're interested in finding σ^2 and σ of $Z = X + Y$. Our starting point is:

$$\text{Var}(Z) = \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) = 2 \text{Var}(X)$$

where we made use of the fact that X and Y are independent events (this allowed us to decompose $\text{Var}(X + Y)$ without a covariance term.) So now we've reduced the problem to computing the variance of rolling a single die.

$$\text{Var}(X) = \langle X^2 \rangle - \langle X \rangle^2$$

Computing both parts:

$$\langle X^2 \rangle = \frac{1}{6} [36 + 25 + 16 + 9 + 4 + 1] = \frac{91}{6}$$

$$\langle X \rangle^2 = \left(\frac{1}{6} [6 + 5 + 4 + 3 + 2 + 1] \right)^2 = \left(\frac{21}{6} \right)^2 = \frac{49}{4}$$

So:

$$\sigma_z^2 = \text{Var}(Z) = 2 \text{Var}(X) = 2 \left[\frac{91}{6} - \frac{49}{4} \right] = \frac{35}{6}$$

$$\sigma_z = \sqrt{\frac{35}{6}}$$