

A possible experimental test of quantized gravity

P. J. SALZMAN* and S. CARLIP†
*Department of Physics, University of California
Davis, CA 95616, USA*

Abstract

While it is widely believed that gravity should ultimately be treated as a quantum theory, there remains a possibility that general relativity should not be quantized. If this is the case, the coupling of classical gravity to the expectation value of the quantum stress-energy tensor will naturally lead to nonlinearities in the Schrödinger equation. By numerically investigating time evolution in the nonrelativistic “Schrödinger-Newton” approximation, we show that such nonlinearities may be observable in the next generation of molecular interferometry experiments.

*email: p@dirac.org

†email: carlip@physics.ucdavis.edu

1. Introduction

The first attempts to quantize general relativity appeared in the early 1930s, and in the seventy years that have followed, we have learned a great deal about quantum field theory and gravity. But despite the hard work of many outstanding physicists, a complete, consistent theory of quantum gravity still seems distant [1]. Given the severe difficulties, one might reasonably ask whether the whole project could be a blind alley. Perhaps our prejudice that everything in Nature should be quantized is simply wrong; perhaps general relativity, a theory of spacetime, is fundamentally different from theories of fields within spacetime.

Somewhat surprisingly, the possibility that gravity is essentially classical has not yet been excluded. The simplest model of classical general relativity coupled to quantum matter, sometimes called “semiclassical gravity,” was proposed forty years ago by Møller [2] and Rosenfeld [3]. The Einstein field equations become

$$G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle, \tag{1.1}$$

where the operator-valued stress-energy tensor is replaced by an expectation value. As a Hartree-like approximation to quantized gravity, such a system certainly makes sense. But as Kibble and Randjbar-Daemi stressed [4], viewed as a fundamental theory, such a model leads to nonlinearities in quantum mechanics: the Schrödinger equation for the wave function $|\psi\rangle$ depends on the metric, which in turn depends, through (1.1), on $|\psi\rangle$.^{*} While more complicated models are possible, any such theory must couple classical gravity to quantum sources, and it is hard to see how to do so without introducing similar nonlinearities.

While the literature contains a number of criticisms of semiclassical gravity [4,6–9], none seems decisive [3,5,10,11]. For example, one might argue that measurements with nonquantized gravitational waves could violate the uncertainty principle for quantized matter [7]. But there are intrinsic limitations to the measurement of even a classical gravitational field [5,12]; it is possible, for instance, that the necessary measurement would require an apparatus massive enough to collapse into a black hole [11]. On the experimental side, neutron interferometry [13] and microscopic deflection experiments [14] show that quantum matter interacts gravitationally as expected, but these results do not require quantization of the gravitational field itself. More direct experimental tests have been suggested using superpositions in Bose-Einstein condensates [15] or, in principle, gravitational radiation from quantum systems [16], but these are not yet practical. The standard mechanism for the appearance of inhomogeneities in inflationary cosmology is probably incompatible with semiclassical gravity, requiring at least linear quantum fluctuations in the metric [17], but there are certainly other ways to produce the observed inhomogeneities.

Semiclassical gravity is probably excluded in a strict Everett interpretation of quantum mechanics [8], but one would like to have a result that does not depend on measurement

^{*}Dirac was also aware of this: see [5], p. 1.

theory or the interpretation of quantum mechanics. It is therefore important to ask whether the nonlinearities in semiclassical gravity might be experimentally accessible.

The full semiclassical equations (1.1) are extremely difficult to analyze. But for many purposes, the nonrelativistic Newtonian approximation should be sufficient. We are thus led to the ‘‘Schrödinger-Newton equation’’ [18, 19],

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - m\Phi\psi, \quad \nabla^2\Phi = 4\pi Gm|\psi|^2, \quad (1.2)$$

a Schrödinger equation for matter coupled to a classical gravitational potential that has as its source the expectation value of the mass density. This system has been studied in the past [20–23], and a fair amount is known about the lowest eigenfunctions and eigenvalues, but time evolution remains much more poorly understood [24–26].

In the remainder of this paper, we will investigate the Schrödinger-Newton evolution of an initial Gaussian wave packet. Qualitatively, we will show that self-gravitation can slow or stop the spreading of the wave packet. We find that despite the weakness of gravity, the resulting suppression of interference may be observable in the next generation of matter interferometry experiments [27].

2. Setting up the problem

Let us first consider a few analytic properties of the Schrödinger-Newton equation. Note that although the equation is nonlinear, time evolution preserves the norm of ψ , and a conserved probability current can be written down:

$$\frac{\partial}{\partial t}|\psi|^2 = \vec{\nabla} \cdot \left[\frac{i\hbar}{2m} \left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^* \right) \right]. \quad (2.1)$$

Thus, as in standard quantum mechanics, we can interpret $|\psi|^2$ as a probability density.

We will be interested in an initial Gaussian wave function

$$\psi(r, 0) = \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\alpha r^2/2} \quad (2.2)$$

with width $\alpha^{-1/2}$. In principle, we expect a two-parameter family of solutions, labeled by α and m . But the Schrödinger-Newton equation is invariant under the rescaling

$$m \rightarrow \mu m, \quad \vec{x} \rightarrow \mu^{-3}\vec{x}, \quad t \rightarrow \mu^5 t, \quad \psi \rightarrow \mu^{9/2}\psi, \quad (2.3)$$

so if $\psi(\alpha, m; \vec{x}, t)$ is a solution, so is $\mu^{9/2}\psi(\mu^6\alpha, \mu m; \mu^{-3}\vec{x}, \mu^5 t)$. It therefore suffices to consider a one-parameter family of solutions.

We have investigated the dynamical Schrödinger-Newton equation (1.2) with initial condition (2.2) numerically, using a homegrown PDE solver specifically designed for this problem, along with associated tools to analyze and organize the data. Details may be

found in P. J. Salzman’s dissertation [28], and will appear in a subsequent paper. Briefly, the Schrödinger equation can be discretized by considering the action of a time evolution operator on the known wave function at some time t to yield the unknown wave function at some later time t' . For example, two common schemes are the “explicit method,” which translates the wave function forward in time,

$$\psi(r, \Delta t) = e^{-i\hat{H}\Delta t/\hbar}\psi(r, 0),$$

and the “implicit method,” which translates the wave function backward in time,

$$e^{+i\hat{H}\Delta t/\hbar}\psi(r, \Delta t) = \psi(r, 0).$$

Here \hat{H} is the Schrödinger-Newton Hamiltonian, which involves both derivatives and the integral

$$I_{SN} = \iiint_{\text{all space}} \frac{|\psi(\vec{r}', t)|^2}{|\vec{r} - \vec{r}'|} d^3r', \quad (2.4)$$

the solution to the Poisson equation (1.2). With either scheme, one can perform a Taylor expansion of the energy operators to any desired order, requiring successively higher order derivatives that can be approximated and converted into a numerical algorithm.

Each of these schemes, however, has properties which make it undesirable to use for a numerical PDE solver. The explicit scheme, while simple and computationally inexpensive, can be shown to be numerically unstable. The implicit scheme, while numerically stable, requires that we find an inverse operator, which is complicated and computationally expensive. Additionally, neither scheme is unitary.

We chose to use Cayley’s form, which is an average of the explicit and implicit methods:

$$e^{i\hat{H}\Delta t/2\hbar}\psi(r, \Delta t/2) = e^{-i\hat{H}\Delta t/2\hbar}\psi(r, -\Delta t/2). \quad (2.5)$$

Cayley’s form is unitary and is of higher order than either the implicit or explicit method. The evolution operator on either side of Cayley’s form was Taylor expanded to first order, giving a second order accurate algorithm. Derivative terms were numerically approximated, and yielded a tridiagonal system of linear equations from which the wavefunction at the next timestep could be computed. The integral I_{SN} technically involves the wave function at the “current timestep,” at which it is unknown; we linearized the integral by using the known wave function at the previous timestep. Accuracy was ensured by successively decreasing the timestep Δt and looking at the limiting behavior as Δt approached machine precision.

A wide assortment of techniques was employed to further test the accuracy of our method. As one important benchmark, for each set of input parameters the program was rerun with the potential turned off, and the results were compared with the exact analytic solution for the free particle; the differences were found to be negligible.

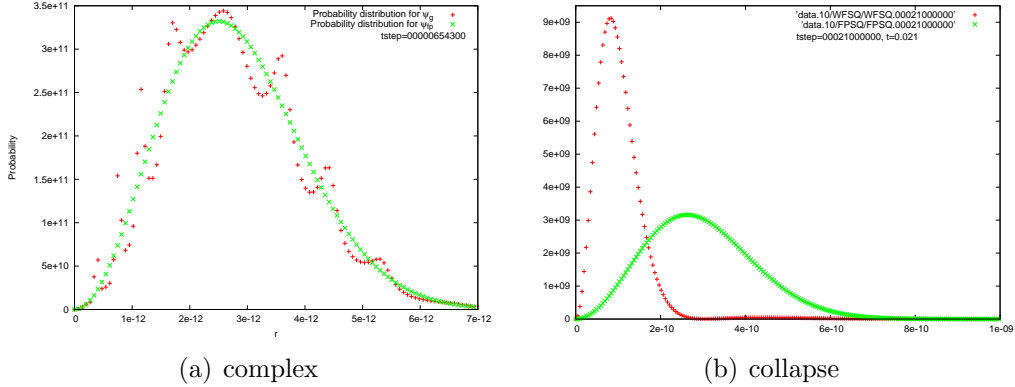


Figure 1: Comparison of Schrödinger-Newton and free particle wave functions. For intermediate masses (a), the evolution becomes complex; for larger values (b), the wave function “collapses.”

3. Numerical Results

The Schrödinger-Newton equation was repeatedly solved using a fixed wave packet width $\alpha = 5 \times 10^{16} \text{ m}^{-2}$ while varying the mass m . The results may be summarized as follows (with masses in unified atomic mass units):

1. For small masses ($< 4.5 \text{ u}$), the behavior is essentially indistinguishable from that of a free particle. As m increases, the wave packet spreads more slowly, as one might expect from “self-gravitation.”
2. For masses between $1.9 \times 10^3 \text{ u}$ and $7.2 \times 10^3 \text{ u}$, the behavior is complex; the wave packet typically fluctuates rapidly and develops growing oscillations (figure 1a). This seems to be related to the behavior found in [24–26].
3. For masses between $7.8 \times 10^3 \text{ u}$ and $2.9 \times 10^{14} \text{ u}$, the wave packet “collapses,” shrinking in width (figure 1b).
4. For larger masses, the wave packet appears stationary; we have not been able to run the program long enough to determine the behavior.

One can obtain a crude estimate of the “collapse” mass by noting that for a free particle, the peak probability density of a Gaussian wave packet occurs at $r_p \sim \alpha^{-1/2} \left(1 + \frac{\alpha^2 \hbar^2}{m^2} t^2\right)^{1/2}$, “accelerating” at a speed $\ddot{r}_p \sim \hbar^2/m^2 r_p^3$. Equating this with the acceleration due to gravity at $t = 0$ yields a mass

$$m \sim \left(\frac{\hbar^2 \sqrt{\alpha}}{G}\right)^{1/3} \sim 10^{10} \text{ u}, \quad (3.1)$$

lying roughly at the middle of the mass range for which we see “collapse.” This expression behaves properly under the scaling (2.3), and could be guessed by dimensional analysis; a

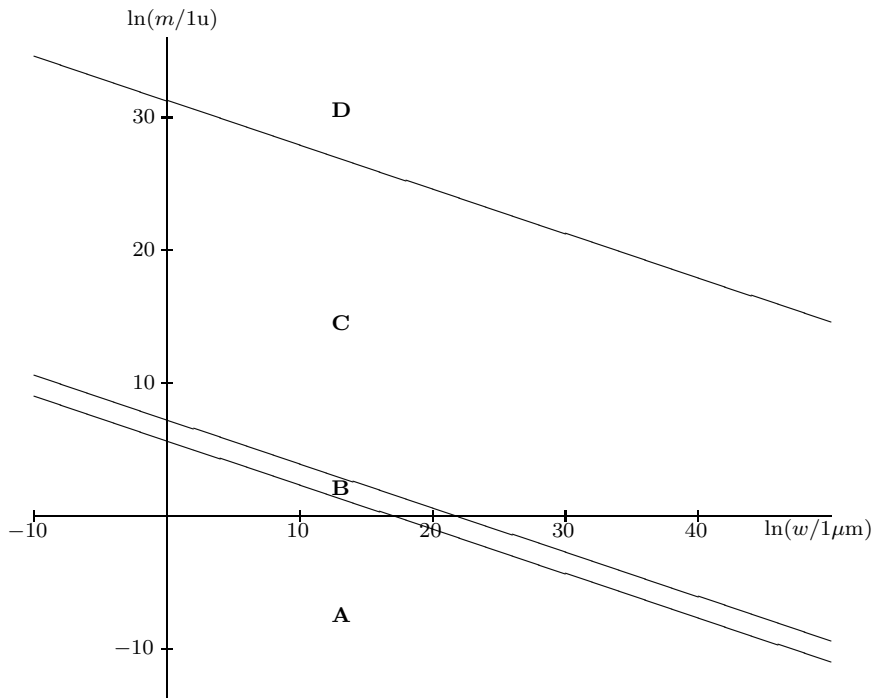


Figure 2: “Phase diagram” for Schrödinger-Newton solutions. Region A: wave packets spread; region B: complex behavior; region C: collapse; region D: undetermined by our simulation.

key result of our numerical simulations is that onset of “collapse” occurs at significantly smaller masses, presumably reflecting the nonlinearity of the evolution.

Using the scaling (2.3), we can summarize our results by the “phase diagram” shown in figure 4. Of particular interest for experiment is the “collapsing” phase (C). As the figure illustrates, the wave packet of fixed width $w = \alpha^{-1/2}$ will shrink in width if its initial mass lies within a range $m_-(w) < m < m_+(w)$. Numerically, with w in microns and m in unified atomic mass units, we find

$$m_-/1 \text{ u} = 1300(w/1 \mu\text{m})^{-1/3}, \quad m_+/1 \text{ u} = 4.8 \times 10^{13}(w/1 \mu\text{m})^{-1/3}. \quad (3.2)$$

The characteristic collapse times in nanoseconds, obtained numerically and scaled according to (2.3), are

$$T_-/1 \text{ ns} = 1.2 \times 10^{-4}(w/1 \mu\text{m})^{-5/3} \quad T_+/1 \text{ ns} = 1.2 \times 10^{-2}(w/1 \mu\text{m})^{-5/3} \quad (3.3)$$

4. Experimental tests

A “collapsing” wave packet will lead to a suppression of interference, which should in principle be observable. Most recent experiments in matter-wave diffraction have used Talbot-Lau interferometry [29], in which an image of a diffraction grating with slit spacing d appears at a distance $L_T = d^2/\lambda$ from the grating. The heaviest molecule for which interference has been observed to date is fluorofullerene, $\text{C}_{60}\text{F}_{48}$, with a mass of 1632 u [30]. The grating slits in this experiment have a width $w \sim .5 \mu\text{m}$. From (3.2), semiclassical gravity would predict a loss of interference for a wave packet of this width for masses greater than $m \sim 1600 \text{ u}$; $\text{C}_{60}\text{F}_{48}$ lies just at the edge of this range.

This is, of course, an oversimplification: the molecular wave packets in [30] are not spherically symmetric Gaussians. The grating slits limit the width in one direction, say y , while the widths in the x and z directions are determined by other factors. Repeating the argument that led to (3.1) for a cylinder and a slab, we might expect a new limiting mass on the order of

$$m'_- = m_-(w_x/w)^{1/3}(w_z/w)^{1/3} \quad (4.1)$$

In the fluorofullerene experiment, w_z was controlled by a height limiter with a width of $100\ \mu\text{m}$, giving a factor of about 6 in (4.1). The appropriate value for the width in the direction of the beam is less clear. As a pessimistic estimate, we note that the distance from the first grating, which is responsible for the transverse coherence of the beam, and the second grating, responsible for the interference, was .38 m; using this value for w_x in (4.1) would give a factor of about 90. Thus while the results for fluorofullerene interference probably do not directly probe semiclassical gravity, they appear to come within two to three orders of magnitude of a real test.

To proceed further, we need both experimental and theoretical work. On the theory side, simulations should be done with more realistic wave packet profiles. It is also important to see how sensitive our results are to the exact (Gaussian) form of the initial wave function. In particular, since a stable spherically symmetric ground state exists [24], *some* initial profiles must have different behavior than what we have seen.

On the experimental side, some progress can come from using narrower slits and from controlling the longitudinal width w_x , perhaps with a shutter to restrict the time each molecule enters the apparatus. The most useful gain, though, would probably come from measurements of higher mass molecules. This is not an unreasonable goal: experimentalists have argued that by using optical gratings, it may be possible to exhibit interference for molecules with masses of up to $10^6\ \text{u}$ [27, 29, 31, 32]. If the next generation of matter wave interferometers can come even close to this limit, an unambiguous test of semiclassical gravity should be possible.

Acknowledgments

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